Demostrar que $\lim\limits_{x \to 5} 3x-8 = 7$

P.D. $\forall \varepsilon>0 \ \exists \delta>0 \ \forall x \in D_f \ (0<|x-5|<\delta \Rightarrow |3x-8-7|<\varepsilon)$

Sea $\varepsilon>0$. Definimos $\delta = \frac{\varepsilon}{3} \Rightarrow \delta>0$.

Sea $x \in D_f$ tal que $0<|x-5|<\delta$

$\Rightarrow |3x-8-7| = |3x-15| = |3(x-5)| = |3|\cdot|x-5| < 3\delta = 3\left(\frac{\varepsilon}{3}\right) = \varepsilon$

$\Rightarrow |3x-8-7| < \varepsilon$.

$\therefore \lim\limits_{x \to 5} 3x-8 = 7$

Demostrar que $\lim\limits_{x \to -3} 2x^2+5x+1 = 4$

P.D. $\forall \varepsilon>0 \ \exists \delta>0 \ \forall x \in D_f \ (0<\underbrace{|x-(-3)|}_{|x+3|}<\delta \Rightarrow |2x^2+5x+1-4|<\varepsilon)$

$\forall a,b \in \mathbb{R}$:

$$\boxed{\begin{array}{c} |a|<b \\ \Updownarrow \\ -b<a<b \end{array}}$$

Notemos que

(*) $\boxed{\text{si } |x+3|<1 \Rightarrow -1<x+3<1 \Rightarrow -4<x<-2 \Rightarrow -8<2x<-4 \Rightarrow -9<2x-1<-5<9 \Rightarrow |2x-1|<9.}$

Sea $\varepsilon>0$. Definimos $\delta = \min\{1, \frac{\varepsilon}{9}\}$. Como $1>0$ y $\frac{\varepsilon}{9}>0 \Rightarrow \delta>0$.

Además, $\delta \leq 1$ y $\delta \leq \frac{\varepsilon}{9}$.

$$\boxed{\begin{array}{l} \text{Si } 0\leq a<b \\ \text{y } 0\leq c<d, \\ \text{entonces} \\ \quad ac<bd \end{array}}$$

Sea $x \in D_f$ tal que $0<|x-(-3)|<\delta \Rightarrow |x+3|<1$ y $|x+3|<\frac{\varepsilon}{9}$.

Como $|x+3|<1 \overset{(*)}{\Rightarrow} |2x-1|<9$

$\Rightarrow |2x^2+5x+1-4| = |2x^2+5x-3| = |(x+3)(2x-1)| = |x+3|\cdot|2x-1| < \left(\frac{\varepsilon}{9}\right)(9) = \varepsilon$

$\Rightarrow |2x^2+5x+1-4| < \varepsilon$.

$\therefore \lim\limits_{x \to -3} 2x^2+5x+1 = 4$